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Chapter 16: Averages

Topic-wise short notes with solved examples

The mean of a set of numbers, how the average changes when numbers are added or removed, and the weighted average of two groups.

1 The average

- The **average** (mean) of n numbers is $\frac{\text{sum of the numbers}}{n}$.
- So **sum** = average $\times n$. This is the most useful form: from an average you can always recover the **sum**.
- The average always lies between the smallest and the largest number.
- For equally spaced numbers (like 3, 5, 7, 9, 11) the average is the **middle** number, or the mean of the first and last.

Example 1.1 Find the average of 12, 15, 18, 21 and 24.

SOLUTION

The numbers are equally spaced, so the average is the middle number. Check:

$$\frac{12 + 15 + 18 + 21 + 24}{5} = \frac{90}{5}.$$

18

Example 1.2 Find the average of the first 20 natural numbers.

SOLUTION

The sum is $\frac{20 \times 21}{2} = 210$. The average is $\frac{210}{20}$.

10.5

2 When a number is added, removed or replaced

- Work with **sums**. Find the old sum and the new sum, then take the difference.

Example 2.1 The average of 5 numbers is 20. A sixth number is added and the average becomes 22. Find the sixth number.

SOLUTION

Old sum = $5 \times 20 = 100$. New sum = $6 \times 22 = 132$.

The sixth number is $132 - 100$.

32

Example 2.2 The average age of 8 people is 40 years. A person aged 32 leaves and a new person joins, and the average becomes 41. Find the age of the new person.

SOLUTION

Old sum = 320. New sum = $8 \times 41 = 328$. The sum has gone up by 8.

New person = $32 + 8$.

40 years

Example 2.3 A student scored 70, 75, 80 and 85 in four tests. What must the student score in the fifth test to have an average of 80?

SOLUTION

Needed total = $5 \times 80 = 400$. Present total = 310.

90

3 Average of two groups

- If group A has a members with average m_A and group B has b members with average m_B , the combined average is

$$\frac{a m_A + b m_B}{a + b}$$

- This is a **weighted average**. It lies closer to the average of the **larger** group.

Example 3.1 A class of 30 students has an average mark of 60 and another class of 20 students has an average mark of 70. Find the average of all 50 students.

SOLUTION

Total marks = $30 \times 60 + 20 \times 70 = 1800 + 1400 = 3200$.

Average = $\frac{3200}{50}$.

64

REMEMBER

Never average two averages directly when the groups are of different sizes. Convert each to a total first.