

OLYMPIAD · MENTAL ABILITY · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 9: Clocks and Calendars

Topic-wise short notes with solved examples

The angle between the hands of a clock, when the hands meet, and how to find the day of the week from odd days.

1 The hands of a clock

- A clock face is a full circle of 360° with 12 hours, so each hour mark is 30° apart and each minute mark is 6° apart.
- The **minute hand** turns 6° every minute. The **hour hand** turns 0.5° every minute (30° in one hour).
- So the minute hand turns 5.5° more than the hour hand every minute.
- The angle between the hands at h hours and m minutes is

$$\theta = |30h - 5.5m|$$

- If the answer is more than 180° , subtract it from 360° to get the smaller angle.

Example 1.1 Find the angle between the hands at 4:30.

SOLUTION

$$\theta = |30 \times 4 - 5.5 \times 30| = |120 - 165| = 45.$$

45°

Example 1.2 Find the angle between the hands at 7:20.

SOLUTION

$$\theta = |30 \times 7 - 5.5 \times 20| = |210 - 110| = 100.$$

100°

2 When the hands meet

- In every 12 hours the two hands **coincide 11 times** (not 12), and are **opposite** 11 times.
- In 24 hours the hands coincide 22 times and are at right angles 44 times.
- The hands coincide between h and $h + 1$ at $\frac{60h}{11}$ minutes past h .

Example 2.1 At what time between 4 and 5 do the hands coincide?

SOLUTION

$$\frac{60 \times 4}{11} = 21\frac{9}{11} \text{ minutes past 4.}$$

4:21 and $\frac{9}{11}$ minutes

Example 2.2 How many times do the hands of a clock coincide in one full day (24 hours)?

SOLUTION

11 times in every 12 hours.

22 times

3 Odd days and the calendar

- A week has 7 days, so any number of days leaves a remainder when divided by 7. That remainder is the number of **odd days**.
- An **ordinary year** has 365 days = 52 weeks + **1 odd day**. A **leap year** has 366 days = 52 weeks + **2 odd days**.
- A year is a leap year if it is divisible by 4. A century year (1900, 2000, 2100) is a leap year only if it is divisible by **400**. So 2000 is a leap year, but 1900 and 2100 are not.

Month	Days	Month	Days
January	31	July	31
February	28 or 29	August	31
March	31	September	30
April	30	October	31
May	31	November	30
June	30	December	31

- To find a day, count the days between the two dates, divide by 7, and move forward by the remainder.

Example 3.1 5 March of a year is a Wednesday. What day of the week is 26 March of the same year?

SOLUTION

From 5 March to 26 March is 21 days. $21 = 7 \times 3$, so the remainder is 0.

Wednesday

Example 3.2 Today is Monday. What day will it be after 100 days?

SOLUTION

$100 = 7 \times 14 + 2$, so move 2 days forward.

Wednesday

Example 3.3 A month of 31 days begins on a Tuesday. How many Tuesdays does it have?

SOLUTION

$31 = 4 \times 7 + 3$. The first three days of the week (Tuesday, Wednesday, Thursday) occur 5 times.

5

REMEMBER

In a month the extra days (28, 29, 30 or 31 days minus 28) are the first few days of the week of that month. Those weekdays occur five times.