

CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 9: HOTS: Applications of Trigonometry (Class 10)

Topic-wise short notes with solved examples

Higher order thinking questions on heights and distances for Class 10: two angles of elevation or depression, towers and buildings, and objects on opposite sides, each with a full solution.

1 Solved HOTS questions

- Draw a clear figure first and mark the horizontal line. The angle of elevation or depression is measured from the horizontal. Useful values: $\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\tan 45^\circ = 1$, $\tan 60^\circ = \sqrt{3}$.

Example 1.1 From the top of a building 60 m high, the angles of depression of the top and the bottom of a tower are 30° and 60° . Find the height of the tower.

SOLUTION

$$\text{Horizontal distance } d = \frac{60}{\tan 60^\circ} = \frac{60}{\sqrt{3}} = 20\sqrt{3} \text{ m.}$$

$$\text{The top of the tower is } d \tan 30^\circ = 20\sqrt{3} \times \frac{1}{\sqrt{3}} = 20 \text{ m below the observer.}$$

$$\text{Height of the tower} = 60 - 20.$$

40 m

Example 1.2 From a point on the ground the angles of elevation of the bottom and the top of a tower fixed at the top of a 30 m high building are 45° and 60° . Find the height of the tower.

SOLUTION

$$\text{The distance from the point to the building is } \frac{30}{\tan 45^\circ} = 30 \text{ m.}$$

$$\text{The top of the tower is at height } 30 \tan 60^\circ = 30\sqrt{3} \text{ m.}$$

$$\text{Height of the tower} = 30\sqrt{3} - 30 = 30(\sqrt{3} - 1).$$

$30(\sqrt{3} - 1)$ m, about 22 m

Example 1.3 The angle of elevation of the top of a tower from a point on the ground is 30° . After walking 20 m towards the tower the angle of elevation becomes 60° . Find the height of the tower.

SOLUTION

Let the height be h and the new distance from the foot be x . Then $\frac{h}{x} = \tan 60^\circ = \sqrt{3}$ and

$$\frac{h}{x+20} = \tan 30^\circ = \frac{1}{\sqrt{3}}.$$

$h = \sqrt{3}x$ and $\sqrt{3}h = x + 20$. Substituting: $3x = x + 20$, so $x = 10$.

$$h = 10\sqrt{3}.$$

$10\sqrt{3}$ m, about 17.3 m

Example 1.4 Two poles of equal height stand on either side of a road 80 m wide. At a point on the road between them the angles of elevation of the tops are 60° and 30° . Find the height of the poles and the distances of the point from the poles.

SOLUTION

Let the distances be x and $80 - x$ and the common height h . $h = x \tan 60^\circ = \sqrt{3}x$ and $h = (80 - x) \tan 30^\circ = \frac{80 - x}{\sqrt{3}}$.

$$3x = 80 - x, \text{ so } x = 20 \text{ and } h = 20\sqrt{3}.$$

Height $20\sqrt{3}$ m (about 34.6 m); distances 20 m and 60 m

Example 1.5 A man on the top of a 75 m high lighthouse sees two ships in the same direction due west, with angles of depression 30° and 45° . Find the distance between the ships.

SOLUTION

The nearer ship is $\frac{75}{\tan 45^\circ} = 75$ m from the foot. The farther ship is $\frac{75}{\tan 30^\circ} = 75\sqrt{3}$ m from the foot.

$$\text{The distance between them is } 75\sqrt{3} - 75 = 75(\sqrt{3} - 1).$$

$75(\sqrt{3} - 1)$ m, about 55 m