

CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 5: HOTS: Arithmetic Progressions (Class 10)

Topic-wise short notes with solved examples

Higher order thinking questions on arithmetic progressions for Class 10: sums of multiples, relations between terms, finding an AP from two sums and three numbers in AP, each with a full solution.

1 Solved HOTS questions

- For an AP with first term a and common difference d : $a_n = a + (n - 1)d$ and $S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} (a + l)$. A condition on terms usually gives one equation in a and d .

Example 1.1 Find the sum of all two-digit numbers which are divisible by 7.

SOLUTION

The numbers are 14, 21, 28, ..., 98, an AP with $a = 14$, $d = 7$ and $l = 98$.

$98 = 14 + (n - 1) \times 7$, so $n = 13$.

$$S_{13} = \frac{13}{2} (14 + 98) = \frac{13 \times 112}{2}.$$

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Example 1.2 The 4th term of an AP is 0. Show that its 25th term is three times its 11th term.

SOLUTION

$$a_4 = a + 3d = 0, \text{ so } a = -3d.$$

$$a_{25} = a + 24d = 21d \text{ and } a_{11} = a + 10d = 7d.$$

$$21d = 3 \times 7d.$$

$$a_{25} = 3 a_{11}$$

Example 1.3 The sum of the first 7 terms of an AP is 49 and the sum of the first 17 terms is 289. Find the sum of the first n terms.

SOLUTION

$$S_7 = \frac{7}{2} (2a + 6d) = 49, \text{ so } a + 3d = 7.$$

$$S_{17} = \frac{17}{2} (2a + 16d) = 289, \text{ so } a + 8d = 17.$$

Subtracting: $5d = 10$, so $d = 2$ and $a = 1$.

$$S_n = \frac{n}{2} [2 + (n - 1) \times 2] = n^2.$$

$$S_n = n^2$$

Example 1.4 Three numbers are in AP. Their sum is 21 and their product is 315. Find the numbers.

SOLUTION

Take the numbers as $a - d$, a and $a + d$. The sum is $3a = 21$, so $a = 7$.

The product is $7(49 - d^2) = 315$, so $49 - d^2 = 45$ and $d = \pm 2$.

5, 7, 9 (or 9, 7, 5)

Example 1.5 Ramkali saves Rs 5 in the first week, and then increases her weekly saving by Rs 1.75. In which week will she save Rs 20.75?

SOLUTION

The savings form an AP with $a = 5$ and $d = 1.75$.

$20.75 = 5 + (n - 1) \times 1.75$, so $n - 1 = \frac{15.75}{1.75} = 9$.

The 10th week