

CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 7: HOTS: Coordinate Geometry (Class 10)

Topic-wise short notes with solved examples

Higher order thinking questions on coordinate geometry for Class 10: points on a line, the section formula, collinearity and shapes from their vertices, each with a full solution.

1 Solved HOTS questions

- Tools: distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; section formula for the ratio $m : n$: $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$; midpoint is the case $1 : 1$. A point on the x-axis has $y = 0$, and a point on the y-axis has $x = 0$.

Example 1.1 Find the point on the x-axis which is equidistant from $(2, -5)$ and $(-2, 9)$.

SOLUTION

Let the point be $(x, 0)$. Its distances: $(x - 2)^2 + 25 = (x + 2)^2 + 81$.

$x^2 - 4x + 29 = x^2 + 4x + 85$, so $-8x = 56$ and $x = -7$.

$(-7, 0)$

Example 1.2 Find the ratio in which the line segment joining $(-3, 10)$ and $(6, -8)$ is divided by the point $(-1, 6)$.

SOLUTION

Let the ratio be $k : 1$. The x-coordinate is $\frac{6k - 3}{k + 1} = -1$, so $6k - 3 = -k - 1$ and $k = \frac{2}{7}$.

Check with y: $\frac{-8k + 10}{k + 1} = \frac{-16/7 + 10}{9/7} = \frac{54/7}{9/7} = 6$. It matches.

$2 : 7$

Example 1.3 Find the value of k so that the points $(1, 5)$, $(2, 3)$ and $(-2, k)$ are collinear.

SOLUTION

For three points to be collinear the area of the triangle they form is zero.

$$\frac{1}{2} |1(3 - k) + 2(k - 5) + (-2)(5 - 3)| = 0.$$

$$3 - k + 2k - 10 - 4 = 0, \text{ so } k - 11 = 0.$$

$$k = 11$$

Example 1.4 Show that the points $A(1, 7)$, $B(4, 2)$, $C(-1, -1)$ and $D(-4, 4)$ are the vertices of a square.

SOLUTION

$AB = \sqrt{9 + 25} = \sqrt{34}$, $BC = \sqrt{25 + 9} = \sqrt{34}$, $CD = \sqrt{9 + 25} = \sqrt{34}$, $DA = \sqrt{25 + 9} = \sqrt{34}$. All sides are equal.

The diagonals are $AC = \sqrt{4 + 64} = \sqrt{68}$ and $BD = \sqrt{64 + 4} = \sqrt{68}$. They are equal.

A quadrilateral with four equal sides and equal diagonals is a square.

ABCD is a square

Example 1.5 The centre of a circle is $(2a, a - 7)$. The circle passes through $(11, -9)$ and has radius $5\sqrt{2}$. Find the values of a .

SOLUTION

The distance from the centre to $(11, -9)$ is the radius: $(2a - 11)^2 + (a - 7 + 9)^2 = 50$.

$4a^2 - 44a + 121 + a^2 + 4a + 4 = 50$, so $5a^2 - 40a + 75 = 0$, that is $a^2 - 8a + 15 = 0$.

$(a - 3)(a - 5) = 0$.

$a = 3$ or $a = 5$