

CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 3: HOTS: Pair of Linear Equations (Class 10)

Topic-wise short notes with solved examples

Higher order thinking questions on pairs of linear equations for Class 10: equations that become linear after a substitution, number and age problems, consistency conditions and boats in streams, each with a full solution.

1 Solved HOTS questions

- Two good habits: name the unknowns in the first line, and look for a **substitution** such as $u = \frac{1}{x}$ that turns a hard-looking pair into a linear pair.

Example 1.1 Solve $\frac{2}{x} + \frac{3}{y} = 13$ and $\frac{5}{x} - \frac{4}{y} = -2$.

SOLUTION

Put $u = \frac{1}{x}$ and $v = \frac{1}{y}$. Then $2u + 3v = 13$ and $5u - 4v = -2$.

Multiply the first by 4 and the second by 3: $8u + 12v = 52$ and $15u - 12v = -6$. Adding gives $23u = 46$, so $u = 2$.

Then $3v = 13 - 4 = 9$, so $v = 3$.

$x = \frac{1}{2}$ and $y = \frac{1}{3}$

Example 1.2 The digits of a two-digit number add up to 9. The number obtained by reversing the digits is 27 more than the original number. Find the number.

SOLUTION

Let the tens digit be x and the units digit y . Then $x + y = 9$.

$10y + x = 10x + y + 27$ gives $9y - 9x = 27$, so $y - x = 3$.

Adding $x + y = 9$ and $y - x = 3$ gives $y = 6$ and $x = 3$.

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Example 1.3 Five years ago A was three times as old as B. Ten years from now A will be twice as old as B. Find their present ages.

SOLUTION

Let the present ages be a and b . Then $a - 5 = 3(b - 5)$, so $a = 3b - 10$.

And $a + 10 = 2(b + 10)$, so $a = 2b + 10$.

$3b - 10 = 2b + 10$ gives $b = 20$, and then $a = 50$.

A is 50 years old and B is 20 years old

Example 1.4 For what value of k does the pair $kx + 3y = k - 3$ and $12x + ky = k$ have infinitely many solutions?

SOLUTION

For infinitely many solutions $\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$.

From $\frac{k}{12} = \frac{3}{k}$ we get $k^2 = 36$, so $k = 6$ or $k = -6$.

From $\frac{3}{k} = \frac{k-3}{k}$ we get $k - 3 = 3$, so $k = 6$.

$k = 6$

Example 1.5 A boat goes 30 km upstream and 44 km downstream in 10 hours. It goes 40 km upstream and 55 km downstream in 13 hours. Find the speed of the stream and the speed of the boat in still water.

SOLUTION

Let the boat's speed be b and the stream's speed s (km/h). Put $p = \frac{1}{b-s}$ and $q = \frac{1}{b+s}$.

$30p + 44q = 10$ and $40p + 55q = 13$.

Multiply the first by 4 and the second by 3: $120p + 176q = 40$ and $120p + 165q = 39$. Subtracting gives $11q = 1$, so $q = \frac{1}{11}$.

Then $30p = 10 - 4 = 6$, so $p = \frac{1}{5}$. So $b - s = 5$ and $b + s = 11$.

Boat 8 km/h and stream 3 km/h