

CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 2: HOTS: Polynomials (Class 10)

Topic-wise short notes with solved examples

Higher order thinking questions on polynomials for Class 10: expressions in the zeroes, forming polynomials, finding unknown coefficients and division, each with a full solution.

1 Solved HOTS questions

- For a quadratic $ax^2 + bx + c$ with zeroes α and β : $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$. Most HOTS questions ask for an expression that can be rewritten using these two numbers.

Example 1.1 If α and β are the zeroes of $x^2 - 5x + 6$, find $\alpha^2 + \beta^2$ and $\frac{1}{\alpha} + \frac{1}{\beta}$.

SOLUTION

$$\alpha + \beta = 5 \text{ and } \alpha\beta = 6.$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 12 = 13.$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{5}{6}.$$

$$13 \text{ and } \frac{5}{6}$$

Example 1.2 If α and β are the zeroes of $2x^2 - 7x + 3$, find $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

SOLUTION

$$\alpha + \beta = \frac{7}{2} \text{ and } \alpha\beta = \frac{3}{2}.$$

$$\alpha^2 + \beta^2 = \frac{49}{4} - 3 = \frac{37}{4}.$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{37/4}{3/2}.$$

$$\frac{37}{6}$$

Example 1.3 Find k if one zero of $3x^2 + 8x + k$ is the reciprocal of the other.

SOLUTION

If the zeroes are α and $\frac{1}{\alpha}$, their product is 1.

The product is $\frac{k}{3}$, so $\frac{k}{3} = 1$.

$$k = 3$$

Example 1.4 The sum of the zeroes of $x^2 - (k + 6)x + 2(2k - 1)$ is half of their product. Find k .

SOLUTION

Sum = $k + 6$ and product = $2(2k - 1)$.

$$k + 6 = \frac{1}{2} \times 2(2k - 1) = 2k - 1, \text{ so } k = 7.$$

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Example 1.5 Divide $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$ and find the quotient and the remainder.

SOLUTION

$x^3 - 3x^2 + 5x - 3 \div (x^2 - 2)$: the first term of the quotient is x . Subtract $x(x^2 - 2) = x^3 - 2x$ to get $-3x^2 + 7x - 3$.

The next term is -3 . Subtract $-3(x^2 - 2) = -3x^2 + 6$ to get $7x - 9$.

The degree of $7x - 9$ is less than the degree of the divisor, so we stop.

Quotient $x - 3$, **remainder** $7x - 9$