

# CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

## Chapter 4: HOTS: Quadratic Equations (Class 10)

Topic-wise short notes with solved examples

Higher order thinking questions on quadratic equations for Class 10: equations with fractions, conditions on the roots, and word problems on speed, tanks and numbers, each with a full solution.

### 1 Solved HOTS questions

- A quadratic  $ax^2 + bx + c = 0$  has equal roots when  $D = b^2 - 4ac = 0$ , two different real roots when  $D > 0$ , and no real roots when  $D < 0$ . In word problems always check that each root makes sense (a length or a speed cannot be negative).

**Example 1.1** Solve  $\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3}$ , where  $x \neq 2, 4$ .

SOLUTION

Multiply by  $3(x-2)(x-4)$ :  $3[(x-1)(x-4) + (x-3)(x-2)] = 10(x-2)(x-4)$ .

$3(2x^2 - 10x + 10) = 10x^2 - 60x + 80$ , so  $4x^2 - 30x + 50 = 0$ , that is  $2x^2 - 15x + 25 = 0$ .

$(2x-5)(x-5) = 0$ .

$x = 5$  or  $x = \frac{5}{2}$

**Example 1.2** Find the values of  $k$  for which  $(k+1)x^2 - 2(k-1)x + 1 = 0$  has equal roots.

SOLUTION

$D = 4(k-1)^2 - 4(k+1) = 4(k^2 - 3k)$ .

$D = 0$  gives  $k = 0$  or  $k = 3$ . Both keep  $k+1 \neq 0$ .

$k = 0$  or  $k = 3$

**Example 1.3** A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less. Find the speed.

SOLUTION

Let the speed be  $x$  km/h.  $\frac{360}{x} - \frac{360}{x+5} = 1$ .

$360 \times 5 = x(x+5)$ , so  $x^2 + 5x - 1800 = 0$  and  $(x+45)(x-40) = 0$ .

A speed cannot be negative, so  $x = 40$ .

**40 km/h**

**Example 1.4** Two taps running together fill a tank in  $9\frac{3}{8}$  hours. The tap with the larger diameter takes 10 hours less than the other to fill the tank alone. Find the time each takes.

SOLUTION

$9\frac{3}{8} = \frac{75}{8}$ . Let the larger tap take  $x$  hours and the smaller  $x + 10$  hours.  $\frac{1}{x} + \frac{1}{x+10} = \frac{8}{75}$ .

$75(2x + 10) = 8x(x + 10)$ , so  $8x^2 - 70x - 750 = 0$ , that is  $4x^2 - 35x - 375 = 0$ .

$D = 1225 + 6000 = 7225 = 85^2$ , so  $x = \frac{35 + 85}{8} = 15$  (the negative root is rejected).

**15 hours and 25 hours**

**Example 1.5** The sum of the squares of two consecutive multiples of 7 is 637. Find the multiples.

SOLUTION

Let them be  $7n$  and  $7n + 7$ .  $49n^2 + 49(n + 1)^2 = 637$ , so  $n^2 + (n + 1)^2 = 13$ .

$2n^2 + 2n - 12 = 0$ , so  $n^2 + n - 6 = 0$  and  $(n + 3)(n - 2) = 0$ .

Taking the positive multiples,  $n = 2$ .

**14 and 21**