

CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 1: HOTS: Real Numbers (Class 10)

Topic-wise short notes with solved examples

Higher order thinking questions on real numbers for Class 10: HCF and LCM in real situations, Euclid's algorithm, irrational numbers and decimal expansions, each with a full solution.

1 Solved HOTS questions

- Try each question for five minutes before reading the solution. The hidden first step is usually "turn the story into an HCF or an LCM".

Example 1.1 Three bells ring at intervals of 12, 15 and 20 minutes. They ring together at 8:00 am. When will they next ring together?

SOLUTION

They ring together again after a time that is a multiple of 12, 15 and 20 minutes, so after the LCM.

$12 = 2^2 \times 3$, $15 = 3 \times 5$, $20 = 2^2 \times 5$, so $\text{LCM} = 2^2 \times 3 \times 5 = 60$ minutes.

At 9:00 am

Example 1.2 Show that $5 - 2\sqrt{3}$ is irrational.

SOLUTION

Suppose $5 - 2\sqrt{3} = r$ is rational.

Then $\sqrt{3} = \frac{5-r}{2}$, which is rational, because r is rational.

But $\sqrt{3}$ is irrational. This contradiction shows that the assumption was wrong.

$5 - 2\sqrt{3}$ is irrational

Example 1.3 Use Euclid's algorithm to find the HCF of 1260 and 7344.

SOLUTION

$$7344 = 5 \times 1260 + 1044.$$

$$1260 = 1 \times 1044 + 216.$$

$$1044 = 4 \times 216 + 180.$$

$$216 = 1 \times 180 + 36.$$

$$180 = 5 \times 36 + 0.$$

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Example 1.4 Find the smallest number which leaves remainder 5 when divided by 12, 15 and 18.

SOLUTION

The number minus 5 must be a multiple of 12, 15 and 18, so find their LCM.

$12 = 2^2 \times 3$, $15 = 3 \times 5$, $18 = 2 \times 3^2$, so $\text{LCM} = 2^2 \times 3^2 \times 5 = 180$.

The smallest such number is $180 + 5$.

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Example 1.5 Without long division, say whether $\frac{129}{2^2 \times 5^7 \times 7^5}$ has a terminating decimal expansion.

SOLUTION

A fraction in lowest terms terminates only if its denominator has no prime factors other than 2 and 5.

The denominator contains 7^5 , and $129 = 3 \times 43$ has no factor 7 to cancel it.

It does not terminate. It is a non-terminating repeating decimal.