

CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 12: HOTS: Surface Areas and Volumes (Class 10)

Topic-wise short notes with solved examples

Higher order thinking questions on surface areas and volumes for Class 10: combined solids, melting and recasting, conical heaps, frustums and flowing water, each with a full solution.

1 Solved HOTS questions

- Key idea: when a solid is melted and recast, its **volume** stays the same, so equate the volumes. For combined solids, add the volumes, but for the surface area count only the surfaces that are visible.

Take $\pi = \frac{22}{7}$.

Example 1.1 A toy is a cone of base radius 6 cm and height 8 cm with a hemisphere of the same radius fixed on its flat face, so that the curved surface of the hemisphere is outside. Find the total surface area of the toy in terms of π .

SOLUTION

Slant height of the cone $l = \sqrt{6^2 + 8^2} = 10$ cm.

Curved surface of the cone $= \pi rl = 60\pi$. Curved surface of the hemisphere $= 2\pi r^2 = 72\pi$.

The flat face is hidden, so the total is $60\pi + 72\pi$.

132π square cm

Example 1.2 A metallic sphere of radius 4.2 cm is melted and recast into a cylinder of radius 6 cm. Find the height of the cylinder.

SOLUTION

Volume of the sphere $= \frac{4}{3}\pi(4.2)^3$, and volume of the cylinder $= \pi(6)^2h$.

$$h = \frac{4 \times 4.2^3}{3 \times 36} = \frac{4 \times 74.088}{108} = 2.744.$$

2.744 cm

Example 1.3 A cylindrical bucket 32 cm high with radius 18 cm is filled with sand and emptied on the ground to form a conical heap of height 24 cm. Find the radius and the slant height of the heap.

SOLUTION

$$\pi \times 18^2 \times 32 = \frac{1}{3} \pi r^2 \times 24, \text{ so } r^2 = \frac{324 \times 32 \times 3}{24} = 1296 \text{ and } r = 36 \text{ cm.}$$

$$\text{Slant height} = \sqrt{36^2 + 24^2} = \sqrt{1872} = 12\sqrt{13}.$$

Radius 36 cm and slant height $12\sqrt{13}$ cm, about 43.3 cm

Example 1.4 A metal cuboid 44 cm by 21 cm by 12 cm is melted and recast into a cylinder of radius 14 cm. Find the height of the cylinder.

SOLUTION

$$\text{Volume of the cuboid} = 44 \times 21 \times 12 = 11\,088 \text{ cubic cm.}$$

$$\text{Base area of the cylinder} = \frac{22}{7} \times 14 \times 14 = 616 \text{ square cm.}$$

$$\text{Height} = \frac{11\,088}{616}.$$

18 cm

Example 1.5 A bucket is in the shape of a frustum of a cone. Its top and bottom radii are 28 cm and 21 cm and its height is 15 cm. Find its capacity.

SOLUTION

$$\text{Volume} = \frac{\pi h}{3} (R^2 + r^2 + Rr) = \frac{\pi \times 15}{3} (784 + 441 + 588) = 5\pi \times 1813.$$

$$\text{With } \pi = \frac{22}{7}: 5 \times 22 \times 1813 = 28\,490.$$

28 490 cubic cm, about 28.5 litres