

CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 8: HOTS: Introduction to Trigonometry (Class 10)

Topic-wise short notes with solved examples

Higher order thinking questions on trigonometric ratios and identities for Class 10: finding all ratios from one, proving identities, complementary angles and conditions on sums, each with a full solution.

1 Solved HOTS questions

- Basics: $\sin^2 \theta + \cos^2 \theta = 1$, $1 + \tan^2 \theta = \sec^2 \theta$, $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$, and $\sin(90^\circ - \theta) = \cos \theta$. When one ratio is given, draw a right triangle, find the third side by Pythagoras and read off every ratio.

Example 1.1 If $\tan \theta = \frac{4}{3}$, find the value of $\frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta}$.

SOLUTION

Take a right triangle with opposite side 4, adjacent side 3 and hypotenuse 5. So $\sin \theta = \frac{4}{5}$ and $\cos \theta = \frac{3}{5}$.

Numerator = $\frac{12}{5} + \frac{6}{5} = \frac{18}{5}$. Denominator = $\frac{12}{5} - \frac{6}{5} = \frac{6}{5}$.

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Example 1.2 Show that $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta) = 1$.

SOLUTION

$(1 - \sin \theta)(1 + \sin \theta) = 1 - \sin^2 \theta = \cos^2 \theta$.

$1 + \tan^2 \theta = \sec^2 \theta = \frac{1}{\cos^2 \theta}$.

The product is $\frac{1}{\cos^2 \theta} \times \cos^2 \theta$.

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Example 1.3 If $\sin \theta + \cos \theta = \frac{7}{5}$, find $\tan \theta + \cot \theta$.

SOLUTION

Square both sides: $1 + 2 \sin \theta \cos \theta = \frac{49}{25}$, so $\sin \theta \cos \theta = \frac{12}{25}$.

$\tan \theta + \cot \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{12/25}$.

$\frac{25}{12}$

Example 1.4 If $\sec \theta + \tan \theta = 3$, find $\sin \theta$.

SOLUTION

$$(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = \sec^2 \theta - \tan^2 \theta = 1, \text{ so } \sec \theta - \tan \theta = \frac{1}{3}.$$

$$\text{Adding: } 2 \sec \theta = \frac{10}{3}, \text{ so } \sec \theta = \frac{5}{3}. \text{ Subtracting: } \tan \theta = \frac{4}{3}.$$

$$\sin \theta = \tan \theta \times \cos \theta = \frac{4}{3} \times \frac{3}{5}.$$

$$\frac{4}{5}$$

Example 1.5 Evaluate $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$.

SOLUTION

$$\sin 25^\circ = \cos 65^\circ \text{ and } \cos 25^\circ = \sin 65^\circ, \text{ because } 25^\circ + 65^\circ = 90^\circ.$$

$$\text{The expression is } \cos^2 65^\circ + \sin^2 65^\circ.$$

$$1$$