

CLASS 8 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 1: HOTS: A Square and a Cube (Class 8)

Topic-wise short notes with solved examples

Higher order thinking questions on squares and cubes for Class 8: making a perfect square or cube by multiplying or dividing, square and cube roots by prime factors, and patterns of odd numbers, each with a full solution.

1 Solved HOTS questions

- In a perfect square every prime factor has an **even** power; in a perfect cube every prime factor has a power that is a **multiple of 3**. The sum of the first n odd numbers is n^2 . Between n^2 and $(n+1)^2$ there are exactly $2n$ numbers.

Example 1.1 Find the square root of 7056 by prime factorisation.

SOLUTION

$$7056 = 2^4 \times 3^2 \times 7^2.$$

Take half of each power: $2^2 \times 3 \times 7 = 4 \times 21$.

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Example 1.2 Find the smallest number by which 108 must be multiplied to get a perfect cube. Find the cube root of the product.

SOLUTION

$108 = 2^2 \times 3^3$. The power of 2 is 2, so one more factor 2 is needed.

The product is $216 = 2^3 \times 3^3$, and its cube root is $2 \times 3 = 6$.

Multiply by 2; the cube root is 6

Example 1.3 Find the sum of the first 15 odd numbers without adding them one by one.

SOLUTION

The sum of the first n odd numbers is n^2 .

$$15^2 = 225$$

Example 1.4 Find the cube root of 17 576 using the units digit.

SOLUTION

The units digit of 17 576 is 6, so the cube root ends in 6.

Ignore the last three digits: 17 lies between $2^3 = 8$ and $3^3 = 27$, so the tens digit is 2.

Check: $26^3 = 17\,576$.

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Example 1.5 How many numbers lie between 12^2 and 13^2 ? How many of these are perfect squares?

SOLUTION

There are $2 \times 12 = 24$ numbers between 12^2 and 13^2 .

None of them is a perfect square, because the next square after 12^2 is 13^2 .

24 numbers, none of them a perfect square