

CLASS 8 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 13: HOTS: Exploring Some Geometric Themes (Class 8)

Topic-wise short notes with solved examples

Higher order thinking questions on fractals, solids and nets for Class 8: counting faces, edges and vertices, Euler's relation, patterns in fractals and views of solids, each with a full solution.

1 Solved HOTS questions

- For a solid with V vertices, E edges and F faces, Euler's relation says $V - E + F = 2$. A **net** is a flat pattern that folds into a solid. A **fractal** repeats the same pattern at smaller and smaller scales.

Example 1.1 Count the faces, edges and vertices of a cube and check Euler's relation.

SOLUTION

A cube has 6 faces, 12 edges and 8 vertices.

$$V - E + F = 8 - 12 + 6 = 2.$$

6 faces, 12 edges, 8 vertices; the relation holds

Example 1.2 A solid has 6 vertices and 9 edges. Find the number of faces and name a solid like this.

SOLUTION

$$6 - 9 + F = 2, \text{ so } F = 5.$$

A triangular prism has 2 triangular faces and 3 rectangular faces, 6 vertices and 9 edges.

5 faces; a triangular prism

Example 1.3 A Sierpinski triangle is made by dividing an equilateral triangle into 4 smaller ones and removing the middle one, then repeating on each remaining triangle. How many small triangles are left after 1, 2 and 3 steps?

SOLUTION

After step 1: 3 triangles. Each step multiplies the count by 3.

After step 2: $3 \times 3 = 9$. After step 3: $9 \times 3 = 27$.

3, 9 and 27

Example 1.4 A line segment of length 1 is changed by replacing its middle third by two sides of an equilateral triangle (a Koch step). Find the length after 1 step and after 2 steps.

SOLUTION

In one step the segment is replaced by 4 pieces, each $\frac{1}{3}$ as long, so the length is multiplied by $\frac{4}{3}$.

After 1 step: $\frac{4}{3}$. After 2 steps: $\frac{16}{9}$.

$\frac{4}{3}$ and $\frac{16}{9}$

Example 1.5 What are the front view and the top view of a cylinder standing on its circular base?

SOLUTION

From the front, a cylinder looks like a rectangle.

From above it looks like a circle.

Front view a rectangle, top view a circle