

CLASS 9 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 4: HOTS: Algebraic Identities (Class 9)

Topic-wise short notes with solved examples

Higher order thinking questions on algebraic identities for Class 9: values of expressions from a sum and a product, quick cubes, factorising sums and differences of cubes and symmetric expressions, each with a full solution.

1 Solved HOTS questions

- Identities to use: $(a + b)^2 = a^2 + 2ab + b^2$; $a^2 - b^2 = (a - b)(a + b)$; $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$; $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$; $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$; $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$. When a sum and a product are given, express the target using only those.

Example 1.1 If $a + b = 10$ and $ab = 21$, find $a^2 + b^2$ and $a^3 + b^3$.

SOLUTION

$$a^2 + b^2 = (a + b)^2 - 2ab = 100 - 42 = 58.$$

$$a^3 + b^3 = (a + b)^3 - 3ab(a + b) = 1000 - 3 \times 21 \times 10 = 1000 - 630 = 370.$$

58 and 370

Example 1.2 Using an identity, find the value of 103^3 .

SOLUTION

$$103^3 = (100 + 3)^3 = 100^3 + 3 \times 100^2 \times 3 + 3 \times 100 \times 3^2 + 3^3.$$

$$1\,000\,000 + 90\,000 + 2700 + 27.$$

1 092 727

Example 1.3 Factorise $8x^3 + 27y^3$.

SOLUTION

$$8x^3 + 27y^3 = (2x)^3 + (3y)^3.$$

Using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ with $a = 2x$ and $b = 3y$.

$$(2x + 3y)(4x^2 - 6xy + 9y^2)$$

Example 1.4 If $x + \frac{1}{x} = 5$, find $x^3 + \frac{1}{x^3}$.

SOLUTION

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right).$$

$$125 = x^3 + \frac{1}{x^3} + 15.$$

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Example 1.5 If $a + b + c = 9$ and $ab + bc + ca = 23$, find $a^2 + b^2 + c^2$.

SOLUTION

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca).$$

$$81 = a^2 + b^2 + c^2 + 46.$$

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