

CLASS 9 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 9: HOTS: Propositions and Converses (Class 9)

Topic-wise short notes with solved examples

Higher order thinking questions on propositions, converses and counterexamples for Class 9: deciding whether a statement is true or false, writing converses and testing them, each with a full solution.

1 Solved HOTS questions

- A **proposition** is a statement that is either true or false, but not both. The **converse** of "If P then Q " is "If Q then P ". A statement and its converse can have different truth values. **One counterexample** is enough to show that a statement is false, but examples never prove that a statement is true for all cases.

Example 1.1 Which of these are propositions: (a) $x + 3 = 5$; (b) 7 is a prime number; (c) Is it raining?; (d) the sum of any two odd numbers is even?

SOLUTION

(a) is true for $x = 2$ and false for other x , so it is not a proposition until x is given.

(b) is a true statement. (c) is a question, not a statement. (d) is a statement, and it is true (for example $3 + 5 = 8$; in general $(2m + 1) + (2n + 1) = 2(m + n + 1)$).

(b) and (d) are propositions

Example 1.2 Write the converse of "If a number is divisible by 6, then it is divisible by 3" and decide whether the converse is true.

SOLUTION

The converse is "If a number is divisible by 3, then it is divisible by 6".

The number 9 is divisible by 3 but not by 6, so this is a counterexample.

The converse is false

Example 1.3 Show that the statement "All prime numbers are odd" is false.

SOLUTION

We need one prime number which is not odd.

The number 2 is prime and even.

2 is a counterexample, so the statement is false

Example 1.4 "If a quadrilateral is a rhombus, then its diagonals are perpendicular" is true. Is its converse true?

SOLUTION

The converse: "If the diagonals of a quadrilateral are perpendicular, then it is a rhombus".

A kite with sides 3, 3, 5 and 5 has perpendicular diagonals but its sides are not all equal, so it is not a rhombus.

The converse is false

Example 1.5 "If n is an even number, then n^2 is even." Write the converse and decide whether both are true.

SOLUTION

Statement: if $n = 2m$ then $n^2 = 4m^2 = 2(2m^2)$, which is even. True.

Converse: "If n^2 is even, then n is even". If n were odd, then n^2 would be odd, so n^2 even forces n even. True.

Both are true