

CLASS 9 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 8: HOTS: Sequences and Progressions (Class 9)

Topic-wise short notes with solved examples

Higher order thinking questions on sequences, arithmetic progressions and geometric progressions for Class 9: finding terms and sums, counting terms and spotting the type of a sequence, each with a full solution.

1 Solved HOTS questions

- Arithmetic progression (AP): add the same number d each time; $a_n = a + (n - 1)d$ and $S_n = \frac{n}{2}[2a + (n - 1)d]$. Geometric progression (GP): multiply by the same number r each time; $a_n = ar^{n-1}$ and $S_n = \frac{a(r^n - 1)}{r - 1}$ for $r \neq 1$. Check the differences first, then the ratios.

Example 1.1 The n th term of a sequence is $3n + 2$. Write the first three terms, find the 20th term and say what kind of sequence it is.

SOLUTION

The first three terms: 5, 8, 11.

The 20th term is $3 \times 20 + 2 = 62$.

The difference is always 3, so it is an AP.

5, 8, 11; the 20th term is 62; an AP with $d = 3$

Example 1.2 Find the sum of the first 10 terms of the AP with first term 5 and common difference 3.

SOLUTION

$$S_{10} = \frac{10}{2}[2 \times 5 + 9 \times 3] = 5 \times 37.$$

185

Example 1.3 Find the 5th and the 8th terms of the GP 2, 6, 18, ...

SOLUTION

$a = 2$ and $r = 3$. The 5th term $= 2 \times 3^4 = 162$.

The 8th term $= 2 \times 3^7 = 2 \times 2187$.

162 and 4374

Example 1.4 The sum of the first three terms of a GP is 39 and the common ratio is 3. Find the first term.

SOLUTION

$$a + 3a + 9a = 13a = 39.$$

$$a = 3 \text{ (the terms are 3, 9, 27)}$$

Example 1.5 Say whether the sequence 1, 4, 9, 16, ... is an AP, a GP, or neither, and find the next term.

SOLUTION

The differences are 3, 5, 7, which are not equal, so it is not an AP.

The ratios are 4, 2.25, 1.78, which are not equal, so it is not a GP.

The terms are the squares $1^2, 2^2, 3^2, 4^2$, so the next term is 5^2 .

Neither; the next term is 25