

CLASS 9 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 11: HOTS: The World of Algorithms (Class 9)

Topic-wise short notes with solved examples

Higher order thinking questions on algorithms for Class 9: following a procedure step by step, the greatest common divisor by repeated subtraction and division, counting the number of steps and improving a method, each with a full solution.

1 Solved HOTS questions

- An **algorithm** is a precise list of steps that always ends with the right answer. To trace one, write the values after every step in a small table. Euclid's algorithm for the greatest common divisor: replace the larger number by the remainder on dividing it by the smaller, and repeat until the remainder is 0; the last non-zero number is the GCD.

Example 1.1 Follow the algorithm: "Start with $a = 48$ and $b = 18$. While $b \neq 0$, replace (a, b) by $(b, a \bmod b)$. Output a ." What is the output?

SOLUTION

$$(48, 18) \rightarrow (18, 12) \text{ since } 48 \bmod 18 = 12.$$

$$(18, 12) \rightarrow (12, 6) \text{ since } 18 \bmod 12 = 6.$$

$$(12, 6) \rightarrow (6, 0) \text{ since } 12 \bmod 6 = 0.$$

Now $b = 0$, so the output is $a = 6$.

6 (the GCD of 48 and 18)

Example 1.2 Find the GCD of 252 and 105 using Euclid's algorithm and count the steps.

SOLUTION

$$252 = 2 \times 105 + 42.$$

$$105 = 2 \times 42 + 21.$$

$$42 = 2 \times 21 + 0.$$

GCD 21, in 3 division steps

Example 1.3 An algorithm subtracts the smaller number from the larger again and again until both are equal. Starting with 36 and 24, what is the final number?

SOLUTION

$(36, 24) \rightarrow (12, 24) \rightarrow (12, 12)$.

The numbers are equal, so stop.

12 (the GCD of 36 and 24)

Example 1.4 How many subtractions does the repeated-subtraction method need for the GCD of 100 and 4, and how many divisions does Euclid's method need?

SOLUTION

Repeated subtraction: $100 - 4 = 96$, then 92, and so on until 4, which takes 24 subtractions to reach $(4, 4)$.

Euclid's method: $100 = 25 \times 4 + 0$, which is 1 division.

24 subtractions against 1 division

Example 1.5 Write an algorithm to decide whether a number n is even.

SOLUTION

Step 1: divide n by 2 and find the remainder.

Step 2: if the remainder is 0, output "even"; otherwise output "odd".

The algorithm uses one division and one decision