

CLASS 9 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 3: HOTS: The World of Numbers (Class 9)

Topic-wise short notes with solved examples

Higher order thinking questions on rational and irrational numbers for Class 9: recurring decimals, rationals between two numbers, rationalising denominators and expressions with surds, each with a full solution.

1 Solved HOTS questions

- A rational number can be written $\frac{p}{q}$ with $q \neq 0$, and its decimal terminates or repeats. An irrational number has a decimal that neither terminates nor repeats. To rationalise $\frac{1}{a + \sqrt{b}}$, multiply the top and bottom by $a - \sqrt{b}$, since $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$.

Example 1.1 Express $0.\overline{27}$ (that is $0.272727\dots$) in the form $\frac{p}{q}$.

SOLUTION

Let $x = 0.272727\dots$. Then $100x = 27.272727\dots$

Subtracting: $99x = 27$, so $x = \frac{27}{99}$.

$$\frac{3}{11}$$

Example 1.2 Find three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

SOLUTION

Write both with the same denominator: $\frac{1}{3} = \frac{8}{24}$ and $\frac{1}{2} = \frac{12}{24}$.

The fractions $\frac{9}{24}$, $\frac{10}{24}$ and $\frac{11}{24}$ lie between them.

$$\frac{3}{8}, \frac{5}{12} \text{ and } \frac{11}{24}$$

Example 1.3 Rationalise the denominator of $\frac{1}{\sqrt{5} - \sqrt{3}}$.

SOLUTION

Multiply the top and bottom by $\sqrt{5} + \sqrt{3}$: the denominator becomes $5 - 3 = 2$.

$$\frac{\sqrt{5} + \sqrt{3}}{2}$$

Example 1.4 If $x = 2 + \sqrt{3}$, find $x + \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$.

SOLUTION

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = 2 - \sqrt{3}.$$

$$x + \frac{1}{x} = 4, \text{ and } x^2 + \frac{1}{x^2} = 4^2 - 2.$$

4 and 14

Example 1.5 Given that $\sqrt{5}$ is irrational, show that $3 + 2\sqrt{5}$ is irrational.

SOLUTION

Suppose $3 + 2\sqrt{5} = r$ is rational. Then $\sqrt{5} = \frac{r - 3}{2}$, which is rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

$3 + 2\sqrt{5}$ is irrational