

CLASS 10 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 21: HOTS Questions for Class 10

Topic-wise short notes with solved examples

Higher order thinking questions for Class 10 on real numbers, quadratic equations, progressions, similar triangles, coordinate geometry, trigonometry, circles, mensuration and probability, each with a full solution.

1 How to use these questions

- A HOTS question joins two ideas or hides its first step. Read it twice and ask: "What is really being asked?"
- Try each question for five minutes before reading the solution. Then compare your method, not only your answer.
- If you get stuck, write what is **given**, what is **needed**, and one fact that connects them.

2 Real numbers

Example 2.1 The HCF of two numbers is 9 and their LCM is 360. If one number is 72, find the other number.

SOLUTION

For two numbers, $\text{HCF} \times \text{LCM} = \text{product of the numbers}$.

$$9 \times 360 = 72 \times n, \text{ so } n = \frac{3240}{72}.$$

45

Example 2.2 Find the largest number that divides 245 and 1029 leaving remainder 5 in each case.

SOLUTION

The number divides $245 - 5 = 240$ and $1029 - 5 = 1024$ exactly, so it is their HCF.

$$240 = 2^4 \times 3 \times 5 \text{ and } 1024 = 2^{10}, \text{ so } \text{HCF} = 2^4 = 16.$$

It is larger than the remainder 5, as it must be.

16

3 Quadratic equations

Example 3.1 Find the value of k for which $x^2 + 2(k + 1)x + k^2 = 0$ has equal roots.

SOLUTION

Equal roots need $D = 0$: $4(k + 1)^2 - 4k^2 = 0$.

$4(2k + 1) = 0$, so $k = -\frac{1}{2}$.

$$k = -\frac{1}{2}$$

Example 3.2 The sum of the squares of two consecutive positive odd numbers is 290. Find the numbers.

SOLUTION

Let the numbers be n and $n + 2$. Then $n^2 + (n + 2)^2 = 290$.

$2n^2 + 4n + 4 = 290$, so $n^2 + 2n - 143 = 0$ and $(n + 13)(n - 11) = 0$.

$n = 11$ since n is positive. The numbers are 11 and 13.

Check: $121 + 169 = 290$.

11 and 13

4 Arithmetic progressions

Example 4.1 The sum of the first n terms of an AP is $3n^2 + 5n$. Find its 20th term and common difference.

SOLUTION

The n th term is $a_n = S_n - S_{n-1}$.

$S_n - S_{n-1} = 3n^2 + 5n - [3(n - 1)^2 + 5(n - 1)] = 3(2n - 1) + 5 = 6n + 2$.

$a_{20} = 6 \times 20 + 2 = 122$, and the common difference is 6.

$a_{20} = 122$ and $d = 6$

5 Similar triangles and coordinates

Example 5.1 In triangle ABC, DE is parallel to BC with D on AB and E on AC. AD = 3 cm, DB = 5 cm and the area of triangle ADE is 18 square cm. Find the area of triangle ABC.

SOLUTION

Triangle ADE is similar to triangle ABC, and $AB = 3 + 5 = 8$ cm.

The ratio of areas is the square of the ratio of sides: $\frac{\text{area ADE}}{\text{area ABC}} = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$.

$$\text{Area ABC} = 18 \times \frac{64}{9}.$$

128 square cm

Example 5.2 In what ratio does the x-axis divide the line segment joining $(2, -3)$ and $(5, 6)$? Find the point of division.

SOLUTION

Let the ratio be $k : 1$. The y-coordinate of the dividing point is $\frac{6k - 3}{k + 1}$, and it is 0 on the x-axis.

$6k = 3$, so $k = \frac{1}{2}$, that is, the ratio is 1 : 2.

The x-coordinate is $\frac{5k + 2}{k + 1} = \frac{2.5 + 2}{1.5} = 3$.

1 : 2, at the point (3, 0)

6 Trigonometry

Example 6.1 If $\tan \theta = \frac{3}{4}$, find the value of $\sin \theta \cos \theta$.

SOLUTION

Take a right triangle with opposite side 3, adjacent side 4 and hypotenuse 5.

$$\sin \theta = \frac{3}{5} \text{ and } \cos \theta = \frac{4}{5}.$$

$$\frac{12}{25}$$

Example 6.2 If $\sin \theta + \sin^2 \theta = 1$, find the value of $\cos^2 \theta + \cos^4 \theta$.

SOLUTION

From $\sin \theta + \sin^2 \theta = 1$ we get $\cos^2 \theta = \sin \theta$.

Then $\cos^4 \theta = \sin^2 \theta$, so $\cos^2 \theta + \cos^4 \theta = \sin \theta + \sin^2 \theta$.

1

Example 6.3 From the top of a building 60 m high, the angles of depression of the top and the bottom of a tower are 30° and 60° . Find the height of the tower.

SOLUTION

Horizontal distance to the tower: $d = \frac{60}{\tan 60^\circ} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$ m.

Drop to the top of the tower: $d \tan 30^\circ = 20\sqrt{3} \times \frac{1}{\sqrt{3}} = 20$ m.

Height of the tower = $60 - 20$.

40 m

7 Circles and mensuration

Example 7.1 In triangle ABC, AB = 12 cm, BC = 8 cm and CA = 10 cm. The incircle touches the sides. Find the length of the tangent from A to the circle.

SOLUTION

The semi-perimeter $s = \frac{12 + 8 + 10}{2} = 15$ cm.

The tangent from A has length $s - BC = 15 - 8$.

7 cm

Example 7.2 A circle is drawn inside a square of side 14 cm so that it touches all four sides. Find the area of the part of the square outside the circle (take $\pi = \frac{22}{7}$).

SOLUTION

The radius is 7 cm. Area of the circle = $\frac{22}{7} \times 7 \times 7 = 154$.

Area of the square = 196.

42 square cm

Example 7.3 A metal sphere of radius 6 cm is melted and recast into small cones of radius 2 cm and height 3 cm. How many cones are made?

SOLUTION

Volume of the sphere = $\frac{4}{3}\pi \times 6^3 = 288\pi$.

Volume of one cone = $\frac{1}{3}\pi \times 2^2 \times 3 = 4\pi$.

Number of cones = $\frac{288\pi}{4\pi}$.

72

8 Probability

Example 8.1 Two dice are thrown together. Find the probability that the sum of the numbers is a prime number.

SOLUTION

Prime sums are 2, 3, 5, 7 and 11. The numbers of ways: 1, 2, 4, 6 and 2, which add to 15.

There are 36 equally likely outcomes.

$$\frac{15}{36} = \frac{5}{12}$$

REMEMBER

In a HOTS question, a quantity you did not know you needed (an area, a tangent length, a total) is often the first step. Ask what the unknown depends on.