

CLASS 8 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 19: HOTS Questions for Class 8

Topic-wise short notes with solved examples

Higher order thinking questions for Class 8 on numbers, algebra, money, geometry, mensuration and data, each with a full solution.

1 How to use these questions

- A HOTS question joins two ideas or hides its first step. Read it twice and ask: "What is really being asked?"
- Try each question for five minutes before reading the solution. Then compare your method, not only your answer.
- If you get stuck, write what is **given**, what is **needed**, and one fact that connects them.

2 Numbers and patterns

Example 2.1 Find the smallest number by which 1620 must be multiplied so that the product is a perfect square. Find the square root of the product.

SOLUTION

Prime factorisation: $1620 = 2^2 \times 3^4 \times 5$.

In a perfect square every prime has an even power. Only the 5 has an odd power, so multiply by 5.

The product is $8100 = 2^2 \times 3^4 \times 5^2$, and its square root is $2 \times 9 \times 5 = 90$.

Multiply by 5; the square root is 90

Example 2.2 Find the smallest number by which 392 must be divided so that the quotient is a perfect cube.

SOLUTION

$392 = 2^3 \times 7^2$.

In a perfect cube every power must be a multiple of 3. The power of 7 is 2, so remove $7^2 = 49$.

The quotient is $2^3 = 8$, which is a cube.

49

Example 2.3 The sum of the digits of a two-digit number is 9. If 27 is added to the number, the digits are reversed. Find the number.

SOLUTION

Let the tens digit be a and the units digit b . Then $a + b = 9$.

$10a + b + 27 = 10b + a$, so $9a - 9b = -27$ and $a - b = -3$.

Adding $a + b = 9$ and $a - b = -3$ gives $2a = 6$, so $a = 3$ and $b = 6$.

Check: $36 + 27 = 63$, which is 36 reversed.

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3 Algebra

Example 3.1 The denominator of a fraction is 4 more than its numerator. When 3 is added to both, the fraction becomes $\frac{3}{4}$. Find the fraction.

SOLUTION

Let the numerator be n . The fraction is $\frac{n}{n+4}$.

$\frac{n+3}{n+7} = \frac{3}{4}$, so $4n + 12 = 3n + 21$ and $n = 9$.

The fraction is $\frac{9}{13}$. Check: $\frac{12}{16} = \frac{3}{4}$.

$\frac{9}{13}$

Example 3.2 Factorise $x^4 - 81$ completely.

SOLUTION

$x^4 - 81 = (x^2)^2 - 9^2 = (x^2 - 9)(x^2 + 9)$.

Also $x^2 - 9 = (x - 3)(x + 3)$. The factor $x^2 + 9$ cannot be broken further.

$(x - 3)(x + 3)(x^2 + 9)$

Example 3.3 If $a + \frac{1}{a} = 5$, find $a^2 + \frac{1}{a^2}$.

SOLUTION

Square both sides: $a^2 + 2 + \frac{1}{a^2} = 25$.

So $a^2 + \frac{1}{a^2} = 25 - 2$.

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4 Money and percentage

Example 4.1 A shopkeeper sells two articles for Rs 1200 each. On one he gains 20% and on the other he loses 20%. Find the overall gain or loss.

SOLUTION

The article sold at a gain has cost price $\frac{1200}{1.2} = 1000$.

The article sold at a loss has cost price $\frac{1200}{0.8} = 1500$.

Total cost = 2500 and total selling price = 2400, so there is a loss of Rs 100.

Loss percent = $\frac{100}{2500} \times 100 = 4$.

A loss of Rs 100, which is 4%

Example 4.2 A number is increased by 20% and then the result is decreased by 20%. The final value is 960. Find the original number.

SOLUTION

Factor = $1.2 \times 0.8 = 0.96$.

$0.96 \times \text{original} = 960$, so the original is $\frac{960}{0.96}$.

1000

Example 4.3 The population of a town is 50 000. It grows by 10% in the first year and by 20% in the second year. Find the population after two years.

SOLUTION

$50\,000 \times 1.1 \times 1.2 = 66\,000$.

66 000

5 Geometry and mensuration

Example 5.1 Each interior angle of a regular polygon is 140° . Find the number of sides.

SOLUTION

The exterior angle is $180^\circ - 140^\circ = 40^\circ$.

The exterior angles of any polygon add up to 360° , so the number of sides is $\frac{360}{40}$.

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Example 5.2 The perimeter of a rectangle is 60 m and its length is 6 m more than its breadth. Find its area.

SOLUTION

$l + b = 30$ and $l = b + 6$, so $2b + 6 = 30$ and $b = 12$, $l = 18$.

Area = 18×12 .

216 square metres

Example 5.3 The sides of a cuboid are in the ratio 1 : 2 : 3 and its volume is 48 cubic cm. Find its total surface area.

SOLUTION

Let the sides be x , $2x$ and $3x$. Volume = $6x^3 = 48$, so $x^3 = 8$ and $x = 2$.

The sides are 2, 4 and 6 cm.

Surface area = $2(2 \times 4 + 4 \times 6 + 2 \times 6) = 2(8 + 24 + 12) = 88$.

88 square cm

6 Data and chance

Example 6.1 The mean of 5 numbers is 12. Each number is multiplied by 3 and then 2 is added. Find the new mean.

SOLUTION

Multiplying every number by 3 multiplies the mean by 3: 36.

Adding 2 to every number adds 2 to the mean: 38.

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Example 6.2 A bag has 3 red, 5 blue and 2 green balls. One ball is picked at random. Find the probability that it is not blue.

SOLUTION

Total balls = 10. Balls that are not blue = $3 + 2 = 5$.

Probability = $\frac{5}{10}$.

$\frac{1}{2}$

REMEMBER

Many HOTS questions become easy once you write what you know as an equation. Name the unknown in the first line.