

CLASS 9 · HOTS · OLYMPIAD PREPARATION · SHORT NOTES

Chapter 20: HOTS Questions for Class 9

Topic-wise short notes with solved examples

Higher order thinking questions for Class 9 on number systems, polynomials, linear equations, lines and angles, triangles, quadrilaterals, circles, mensuration and data, each with a full solution.

1 How to use these questions

- A HOTS question joins two ideas or hides its first step. Read it twice and ask: "What is really being asked?"
- Try each question for five minutes before reading the solution. Then compare your method, not only your answer.
- If you get stuck, write what is **given**, what is **needed**, and one fact that connects them.

2 Number systems

Example 2.1 If $x = 3 + 2\sqrt{2}$, find $x + \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$.

SOLUTION

$$\frac{1}{x} = \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}} = \frac{3 - 2\sqrt{2}}{9 - 8} = 3 - 2\sqrt{2}.$$

$$x + \frac{1}{x} = (3 + 2\sqrt{2}) + (3 - 2\sqrt{2}) = 6.$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 36 - 2 = 34.$$

6 and 34

Example 2.2 Simplify $\sqrt{7 + 4\sqrt{3}}$.

SOLUTION

Look for $a + b\sqrt{3}$ with $(a + b\sqrt{3})^2 = a^2 + 3b^2 + 2ab\sqrt{3}$.

We need $2ab = 4$ and $a^2 + 3b^2 = 7$. Try $a = 2$ and $b = 1$: $4 + 3 = 7$ and $2 \times 2 \times 1 = 4$.

So $7 + 4\sqrt{3} = (2 + \sqrt{3})^2$.

$2 + \sqrt{3}$

3 Polynomials

Example 3.1 If $x + \frac{1}{x} = 4$, find $x^3 + \frac{1}{x^3}$.

SOLUTION

Cube both sides: $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} = 64$.

So $x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 64$, which gives $x^3 + \frac{1}{x^3} = 64 - 12$.

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Example 3.2 Factorise $x^3 - 6x^2 + 11x - 6$.

SOLUTION

Try $x = 1$: $1 - 6 + 11 - 6 = 0$, so $(x - 1)$ is a factor.

Dividing gives $x^2 - 5x + 6 = (x - 2)(x - 3)$.

$(x - 1)(x - 2)(x - 3)$

Example 3.3 The polynomial $p(x) = x^3 + ax^2 + bx + 6$ is divisible by both $(x - 1)$ and $(x - 2)$. Find a and b .

SOLUTION

$p(1) = 0$: $1 + a + b + 6 = 0$, so $a + b = -7$.

$p(2) = 0$: $8 + 4a + 2b + 6 = 0$, so $2a + b = -7$.

Subtracting the first equation from the second gives $a = 0$, and then $b = -7$.

Check: $x^3 - 7x + 6 = (x - 1)(x - 2)(x + 3)$.

$a = 0$ and $b = -7$

4 Linear equations

Example 4.1 The sum of a two-digit number and the number formed by reversing its digits is 99. The difference of the digits is 3 and the tens digit is larger. Find the number.

SOLUTION

Let the digits be a (tens) and b (units). The number plus its reverse is $11(a + b) = 99$, so $a + b = 9$.

$a - b = 3$. Adding: $2a = 12$, so $a = 6$ and $b = 3$.

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5 Lines, angles and triangles

Example 5.1 In triangle ABC, the bisectors of angle B and angle C meet at O. If angle A is 70° , find angle BOC.

SOLUTION

$$\angle B + \angle C = 180^\circ - 70^\circ = 110^\circ.$$

$$\text{In triangle BOC: } \angle OBC + \angle OCB = \frac{1}{2}(110^\circ) = 55^\circ.$$

$$\angle BOC = 180^\circ - 55^\circ = 125^\circ. \text{ (In general } \angle BOC = 90^\circ + \frac{1}{2}\angle A.)$$

125°

Example 5.2 In a right-angled triangle the hypotenuse is 25 cm and one leg is 7 cm. Find the length of the median drawn to the hypotenuse.

SOLUTION

The midpoint of the hypotenuse is the centre of the circle through all three vertices, so it is equally far from all of them.

The median is half the hypotenuse.

12.5 cm

6 Quadrilaterals

Example 6.1 In a parallelogram ABCD, angle A is $(2x + 10)^\circ$ and angle B is $(3x - 20)^\circ$. Find angle A.

SOLUTION

Adjacent angles of a parallelogram add up to 180° : $2x + 10 + 3x - 20 = 180$.

$$5x - 10 = 180, \text{ so } x = 38.$$

$$\angle A = 2 \times 38 + 10 = 86.$$

86°

Example 6.2 The perimeter of triangle ABC is 48 cm. D, E and F are the midpoints of its sides. Find the perimeter of triangle DEF.

SOLUTION

By the midpoint theorem each side of triangle DEF is half of the opposite side of triangle ABC.

So the perimeter of DEF is half of 48.

24 cm

7 Circles and mensuration

Example 7.1 A chord of length 16 cm is drawn in a circle of radius 10 cm. Find its distance from the centre.

SOLUTION

The perpendicular from the centre bisects the chord, so a half-chord is 8 cm.

$$d = \sqrt{10^2 - 8^2} = \sqrt{36}.$$

6 cm

Example 7.2 The sides of a triangle are 13 cm, 14 cm and 15 cm. Find the height on the side of length 14 cm.

SOLUTION

$$s = \frac{13 + 14 + 15}{2} = 21.$$

$$\text{Area} = \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84.$$

$$\frac{1}{2} \times 14 \times h = 84, \text{ so } h = 12.$$

12 cm

Example 7.3 The radius of a cone is 7 cm and its height is 24 cm. Find its curved surface area (take $\pi = \frac{22}{7}$).

SOLUTION

$$\text{Slant height } l = \sqrt{7^2 + 24^2} = \sqrt{625} = 25 \text{ cm.}$$

$$\text{Curved surface area} = \pi r l = \frac{22}{7} \times 7 \times 25.$$

550 square cm

8 Statistics

Example 8.1 The mean of 10 numbers is 20. If one number, 25, is replaced by 45, find the new mean.

SOLUTION

$$\text{Old sum} = 200. \text{ New sum} = 200 - 25 + 45 = 220.$$

$$\text{New mean} = \frac{220}{10}.$$

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REMEMBER

Many HOTS questions are two simple facts in a row. Find the first fact (the hidden step), and the second one is usually routine.

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