

OLYMPIAD · EVERYDAY MATHEMATICS · OLYMPIAD PREPARATION · SHORT NOTES

# Chapter 13: Ratio and Proportion

Topic-wise short notes with solved examples

Comparing quantities by ratio, sharing in a ratio, proportion, and direct and inverse variation in everyday problems.

## 1 Ratio

- The **ratio**  $a : b$  compares two quantities of the same kind. It is the fraction  $\frac{a}{b}$  and has no units.
- A ratio is simplified by dividing both terms by their common factor:  $45 : 60 = 3 : 4$ .
- To **share** a quantity in the ratio  $a : b : c$ , add the parts  $a + b + c$ . One part is the quantity divided by this sum; then multiply by  $a$ ,  $b$  and  $c$ .
- If  $a : b$  and  $b : c$  are given, make the value of  $b$  the same in both, then join them to get  $a : b : c$ .

**Example 1.1** Divide Rs 360 in the ratio 2 : 3 : 4.

SOLUTION

Total parts =  $2 + 3 + 4 = 9$ . One part =  $\frac{360}{9} = 40$ .

Shares:  $2 \times 40$ ,  $3 \times 40$ ,  $4 \times 40$ .

**Rs 80, Rs 120 and Rs 160**

**Example 1.2** If  $a : b = 2 : 3$  and  $b : c = 4 : 5$ , find  $a : b : c$ .

SOLUTION

Make  $b$  equal in both:  $a : b = 8 : 12$  and  $b : c = 12 : 15$ .

**8 : 12 : 15**

**Example 1.3** The ages of two people are in the ratio 3 : 5 and their sum is 40 years. Find the ages.

SOLUTION

One part =  $\frac{40}{8} = 5$ .

**15 years and 25 years**

## 2 Proportion

- Four numbers  $a, b, c, d$  are in **proportion** if  $a : b = c : d$ . Then  $a \times d = b \times c$  (product of extremes = product of means).
- The **fourth proportional** to  $a, b, c$  is  $d = \frac{bc}{a}$ . The **mean proportional** between  $a$  and  $b$  is  $\sqrt{ab}$ .

**Example 2.1** Find  $x$  if  $4 : 7 = 12 : x$ .

SOLUTION

$$4x = 7 \times 12 = 84.$$

$$x = 21$$

**Example 2.2** Find the mean proportional between 4 and 9.

SOLUTION

$$\sqrt{4 \times 9} = \sqrt{36}.$$

$$6$$

### 3 Direct and inverse proportion

- If two quantities increase together at the same rate (cost and number of articles), they are in **direct proportion**: their ratio stays the same.
- If one quantity increases when the other decreases so that the **product** stays the same (workers and days), they are in **inverse proportion**.

**Example 3.1** The cost of 5 pens is Rs 40. Find the cost of 12 pens.

SOLUTION

Cost of one pen = 8, so 12 pens cost  $12 \times 8$ .

**Rs 96**

**Example 3.2** 6 workers can finish a job in 12 days. How many days will 9 workers take?

SOLUTION

The work is  $6 \times 12 = 72$  worker-days. Days =  $\frac{72}{9}$ .

**8 days**

#### REMEMBER

Before you calculate, decide in words: "more workers, fewer days" means inverse. "More pens, more cost" means direct.